Reaction Wheel Assisted Locomotion for Legged Robots

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***Abstract*— In this paper, we introduce a Model Predictive Control (MPC) formulation that calculates the ground reaction forces and torques for a quadruped equipped with two reaction wheels. We augment the centroidal dynamics with two additional angular momentum states from the wheels, and we further simplify the dynamics to formulate the problem as a convex quadratic program. With the simplified dynamics, we are able to run a hardware implementation of the MPC with a ten-time-step horizon and close the control loop at 250 Hertz. Experiment and simulation results demonstrate improved attitude stabilization and up to 40 percent reduction in angular error. A video demonstration of some of the experiments is available[1]. Our approach and formulation provide a framework for more involved acrobatic maneuvers for future work.**

***Paper Type* – Original Work**

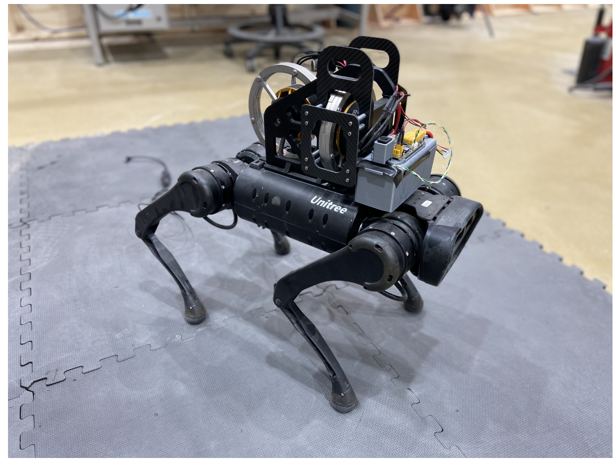

Fig. 1: The Unitree A1 Quadruped mounted with our custom-made reaction wheel module.

I. INTRODUCTION

During dynamic locomotion, legged robots often need to resist sudden and unexpected impacts and disturbances. The disturbance rejection ability of many modern legged robots is fundamentally limited due to the use of point-foot designs. While simple and effective, the point-foot design causes many modern quadrupeds to become underactuated during trotting gait locomotion [3]. During the two-feet standing phase, the robots lose rotation control authority around the line of support, and large body orientation error can only be eliminated by foot mode switching [3, 1].

Our goal is to use novel hardware design to make robot body orientation fully actuated during trotting. Without significantly modifying the standard 12 DOF quadruped robot design, we add a payload module on the back of the robot that provides additional torque control using two reaction wheels. The 5-kg module as shown in Figure 1 is compact, reusable, and designed with high control bandwidth. We use Model Predictive Control (MPC) [2] to control both the robot orientation and wheel speed so the robot orientation stays controllable during a two-leg stance phase. The method, which we call *reaction-wheel MPC*, is validated in both simulation and hardware. The major contributions of this work are:

- Design and construction of a two-axis reaction wheel actuation system.
- A convex MPC algorithm that leverages the reaction wheels to improve disturbance rejection.

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[1]https://youtu.be/fd_mHK6g7vg

II. RELATED WORK

Multiple recent works have focused on external appendages to improve the controllability and mobility of a robot. Libby et. al. designed a comprehensive framework for analyzing planar aerial reorientation for several types of appendages, and they further demonstrated their analysis by designing a planar tail-assisted dynamic self-righting controller [4]. More recently, Norby et. al. took the tail-assisted attitude righting a step further by leveraging air drag with lightweight tails [5].

III. BACKGROUND

A. Centroidal Dynamics

In many implementations of MPC for legged locomotion, the dynamics of the robot are often simplified to a single rigid body with four reaction forces from the ground [1, 2]. The leg masses are ignored with the assumption that they are light enough to be negligible relative to the mass of the torso. Given a robot with center of mass (CoM) position p and inertia I with angular velocity ω, the dynamics of a quadruped under the centroidal model can be written as,

$$\begin{bmatrix} \ddot{p} \\ \frac{d}{dt}(I\omega) \end{bmatrix} = \sum_{i=0}^{n} \begin{bmatrix} \frac{f_i}{m} \\ \hat{r_i} \times f_i \end{bmatrix} - \begin{bmatrix} g \\ 0 \end{bmatrix}, \tag{1}$$

where f_i is the ground reaction force for foot i, r_i is the position of foot i relative to the CoM, and g is gravity. We extend the work done by Di Carlo et al, in which they made several key assumptions to enable the formulation of a linear-time-varying MPC [2]. We leverage the same small-angle approximation around stable walking conditions to linearize pitch and roll dynamics. In addition, we assume that the angular velocity of the body is small enough to leave out the Coriolis term in the rotational dynamics. For the rest of this paper, we will follow the same notation as introduced in the paper by Di Carlo et. al. [2].

IV. GYRO-CENTROIDAL DYNAMICS

Our reaction wheel module adds two reaction wheels that provide body-frame torque controls about the roll and pitch axes. To incorporate the reaction wheels into the control system, we model a quadruped with reaction wheels as a gyrostat. A gyrostat is a system of coupled rigid bodies whose relative motions does not change the total inertia tensor of the system, and the fundamental governing equation that describes this model can be written as,

$$I\dot{\omega} + \omega \times (I\omega + \rho) + \tau_p = \sum_{i=0}^{n} \hat{r}_i \times f_i, \tag{2}$$

where ρ is the total angular momentum stored in the reaction wheels, and τ_ρ is the torque input into the wheels. Using the same small-perturbation assumptions as before, we drop the $\omega \times (I\omega + \rho)$ term and obtain the following dynamics equation

$$I\dot{\omega} + \tau_p = \sum_{i=0}^{n} \hat{r} \times f_i. \tag{3}$$

Combining Equations (1) and (3), we get the following linear-time-varying dynamics for the gyro-centroidal model

$$\frac{d}{dt}\begin{bmatrix}\Theta\\ p\\ \omega\\ \dot{p}\\ \rho\end{bmatrix} = \begin{bmatrix}0_3 & 0_3 & R_z^T(\psi) & 0_3 & 0_3\\ 0_3 & 0_3 & 0_3 & 1_3 & 0_3\\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3\\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3\\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3\end{bmatrix}\begin{bmatrix}\Theta\\ p\\ \omega\\ \dot{p}\\ \rho\end{bmatrix} + \begin{bmatrix}0_3 & \dots & & 0_3\\ 0_3 & \dots & & 0_3\\ I^{-1}[r_1]_\times & \dots & I^{-1}[r_n]_\times & I^{-1}R_z^T(\psi)\\ \frac{1_3}{m} & \dots & \frac{1_3}{m} & 0_3\\ 0_3 & \dots & & 1_3\end{bmatrix}\begin{bmatrix}f_1\\ \vdots\\ f_n\\ \tau_\rho\end{bmatrix} + \begin{bmatrix}0\\ \vdots\\ 0\\ g\end{bmatrix}, \tag{4}$$

where Θ is the robot orientation parameterized by euler angles and $R_z(\psi)$ represents the rotation matrix that is linearized around the yaw angle ψ. The equation can then be written down in a convenient linear-time-varying form,

$$\dot{x}(t) = A(\psi)x(t) + B(r_1, \dots, r_n, \psi)u(t), \tag{5}$$

where A and B are the linearized dynamics matrices about some nominal planned trajectory of ψ and foot positions r_i. $x(t)$ and $u(t)$ are state and control input of the robot at time t.

V. CONVEX MPC FORMULATION

The problem is now linearized with the continuous time transition and control matrices A and B. We convert those matrices into discrete time A_d and B_d matrices. This control problem is then posed as a classic discrete-time linear trajectory optimization problem as follows:

$$\begin{aligned}
\min_{x,u} \quad & \sum_{0}^{k-1} \|x_{i+1}^d - x_{i+1}\|_{Q_i} + \|u_i\|_{R_i} && \text{(6a)}\\
\text{subject to} \quad & x_{i+1} = A_{di}x_i + B_{di}u_i, i = 0 \dots k-1 && \text{(6b)}\\
& c_i \le C_i u_i \le c_i, i = \dots k-1 && \text{(6c)}\\
& Du_i = 0, i = 0 \dots k-1, && \text{(6d)}
\end{aligned}$$

where x_i, u_i, Q_i, R_i are the state of the robot, control inputs to the robot, and cost matrices for state and control inputs at time step i, respectively. The matrices C_i in Equation 6c are used to enforce linearized friction cone constraints for each ground reaction force vector. The equality constraints D_i in Equation 6d are used to constrain foot forces to be zero when a foot is in swing phase. Finally, since we are working with linearized dynamics, the optimization in Equation (6) can be written down as a quadratic program (QP). We also regularize the speed of the reaction wheels inside the dynamics penalty term and constrain the reaction wheel torques with the affine constraints in the QP. The solution of the above MPC problem returns the ground reaction forces for each of the feet in contact with the ground. We convert the ground reaction forces into joint torques as follows,

$$\tau_i = J_i R^T f_i, \tag{7}$$

where τ_i, J_i, f_i, R are the joint torques, forward kinematic jacobian, solved ground reaction forces for leg i, and world frame to body frame rotation matrix, respectively. Swing leg control follows Di Carlo et al's formulation directly.

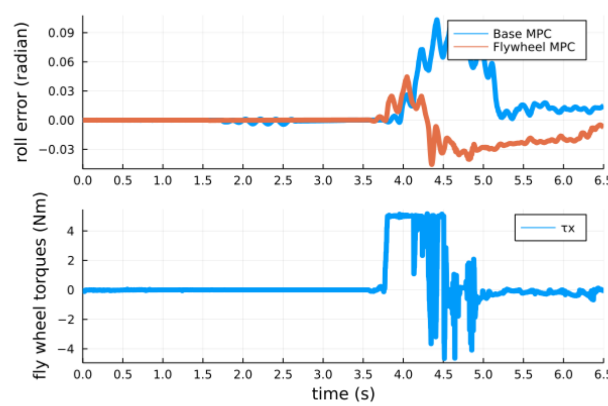
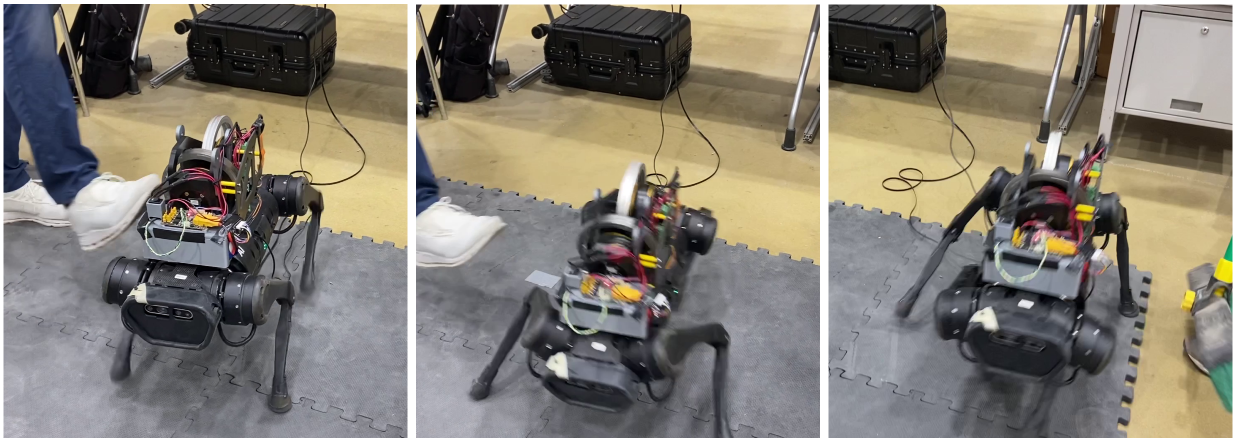


Fig. 2: Robot roll error responses to a 350N impulse on the body frame y-axis at $t = 3.6$ seconds. The top graph illustrated the roll error trajectory with respect to time for the base MPC controller and the reaction wheel assisted controller. The bottom graph plots the torque exerted by the x-axis reaction wheel during the experiment.

VI. SIMULATION RESULTS

We tested the reaction wheel MPC in a controlled Gazebo environment on a modified Unitree A1 model mounted with our reaction wheel module. We performed two kinds of



(a) (b) (c)

Fig. 3: Hardware impulse test where we provide a impulse force on the robot during locomotion with a kick. Figure 3a shows the robot in stable trotting phase when the impulse is applied. Figure 3b shows the robot losing balance on the footholds while maintaining a stable attitude as it eventually recovers from the impulse in Figure 3c.

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experiments on the robot: a disturbance test on the robot body during the trotting phase of locomotion, and an aerial reorientation test where we drop the robot from a specified height at a known attitude offset.

A. Locomotion

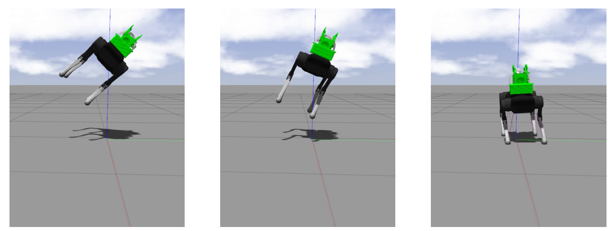
In the disturbance rejection tests, we supplied a 300N, 350N, and 400N impulse on the y axis of the robot body. Figure 2 shows the roll error response of the robot during one of the impact experiments in Gazebo. The experiments demonstrated an enhanced ability to recover from sudden impact. Orientation errors are reduced up to 40 percent in these tests. During the 450N impulse experiments, the reaction wheel enhanced controller consistently recovers from the impact while the base MPC fails.

B. Aerial Re-orientation

In addition to the locomotion test, we also tested the aerial reorientation capability of our reaction wheel add-on module. By locking the joints of the robot and solely relying on the torques from the reaction wheels, we dropped the robot from 0.5m and provide it with angular offsets of 0.6 radians in the pitch axis for the experiment shown in Figure 4. The reaction wheels were able to steer robot and correct its orientation in midair before touchdown. The experiment verifies the reaction wheels are able to quickly correct large orientation errors.

VII. PRELIMINARY HARDWARE RESULTS

We implemented the proposed controller on a Unitree A1 robot with our custom-made payload module, as shown in Figure 1. The baseline controller [2] is as described in [2]. The MPC employed a look-ahead horizon of 0.05 seconds divided into 10 time steps, and the average solve time is 0.004 seconds for a 1.60GHz Intel i5-8250U quad-core CPU. The reaction wheels provided body torque control in the roll and pitch rotation axes, each with a maximum output torque of 5Nm and a maximum spin speed of 2000 RPM. We qualitatively verified that the controller runs as expected and the robot was able to recover from arbitrary impulse disturbances. Figure 3 shows a sequence of the robot maintaining attitude while recovering from an impulse disturbance.



(a) (b) (c)

Fig. 4: A drop test sequence where the robot reorients itself with the torques from the x axis reaction wheel.

VIII. CONCLUSIONS AND FUTURE WORK

In this paper, we demonstrated the feasibility of using reaction wheels to assist attitude control of a quadruped in simulation and hardware testings. For future work, we plan to test the controller on hardware more rigorously and collect more data for analysis. We plan to perform the same kind of controlled impulse rejection test and aerial reorientation test. In addition, the prototype we designed is not optimized for efficiency, and we will analyze its power consumption and optimize in future design iterations. Finally, we will also mathematically analyze the controllability of the quadruped during a two leg stance.

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