Whole Body Nonlinear Model Predictive Control for Legged Robots

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Abstract— **Online planning of whole-body motions for legged robots is challenging due to the inherent nonlinearity in the robot dynamics. To address this limitation, we propose a nonlinear MPC framework, the BiConMP, which can generate whole body trajectories online by efficiently exploiting the structure of the robot dynamics. BiConMP is used to generate various cyclic gaits on a real quadruped robot and its performance is evaluated on different terrain, countering unforeseen pushes and transitioning online between different gaits. Further, we demonstrate the ability of BiConMP to generate non-trivial acyclic whole-body dynamic motions on the robot. Finally, an extensive empirical analysis on the effects of planning horizon and frequency on the nonlinear MPC framework is reported and discussed. "Paper Type – Recent Work [1]".**

I. Introduction

The efficacy of legged robots navigating in human environments depends largely on how efficiently they can move around, adapt to changes in their surroundings and recover from unforeseen disturbances. Since these decisions are usually made by trajectory optimization algorithms, which compute optimal forces and contact planners which decide end-effector interaction timings and locations with the surroundings, it is crucial for these algorithms to run quickly such that they adapt online to dynamic changes in the environment.

Fast trajectory optimization using the whole body robot dynamics is difficult because of the nonlinearity of the problem. Due to which, most existing approaches in this domain make assumptions that either ignore parts of the robot dynamics [2] or restrict the kinds of behaviours the robot can perform [3], [4] with the algorithm in order to gain the ability to update plans online. There are few approaches capable of model predictive control while considering the non linear robot dynamics [5]. However, it is not clear if such frameworks would maintain real time capabilities for highly dynamic behaviours (ex. bounding, flips etc..) which involve angular momentum on the real robot.

We propose a novel nonlinear trajectory optimization framework, The BiconMP [1], that can generate whole body motion plans in real-time (20 Hz) using the kino-dynamic structure proposed in [6]. In the kino-dynamic framework, a feasible centeroidal momentum trajectory is computed based on a given contact plan. After which, an inverse kinematics solver generates a full body joint trajectory satisfying the contact plan, while tracking the centeroidal momentum plan (using the centeroidal momentum matrix). Finally inverse dynamics is used to compute required torques. Since the framework does not make any assumptions on the dynamics it can be used across different legged robots (bipeds, quadrupeds and humanoids). The main contribution of the approach are :

- We propose a novel solver based on ADMM that takes advantage of the biconvex structure of the centroidal dynamics and generates centroidal trajectories efficiently. We also propose to use an efficient method based on proximal operators to solve the convex sub-problems (with second-order cone constraints) which ultimately enables us to perform non linear centroidal MPC.
- To obtain whole-body trajectories, we propose to use a DDP-based kinematic solver (rather than whole-body MPC) to track the centroidal trajectories while penalizing other whole-body constraints. This enables automatic swing foot trajectory generation without the need for hand tuning.
- We implement different cyclic and acyclic gaits on the real Solo12 quadruped along with the transition between these gaits. In particular, we perform a bounding motion on Solo12 with a considerable amount of angular momentum which is not possible to do with state-of-the-art linear MPC approaches.
- We perform extensive analysis on the effects of horizon length and MPC frequency on the performance and robustness of the whole control framework in the presence and absence of different uncertainties and disturbances. To the best of our knowledge, this is the first reported empirical analysis of closed loop non linear MPC for legged robots.

II. Method

The BiConMP is based on the kino-dynamic trajectory generation infrastructure [6], which breaks the whole body optimization problem for under actuated systems into a more tractable centeroidal dynamics and inverse kinematics optimization, which are solved iteratively until convergence.

In the BiConMP, the dynamics optimization problem is solved efficiently by exploiting the biconvex structure in the centroidal dynamics. We use the ADMM algorithm to perform the biconvex optimization as it provides favourable convergence properties such as the ability to reach acceptable solutions in fewer iterations and guaranteed sublinear convergence. Crucially, due to the unconstrained nature of each sub-problem, each iteration is computationally cheap with respect

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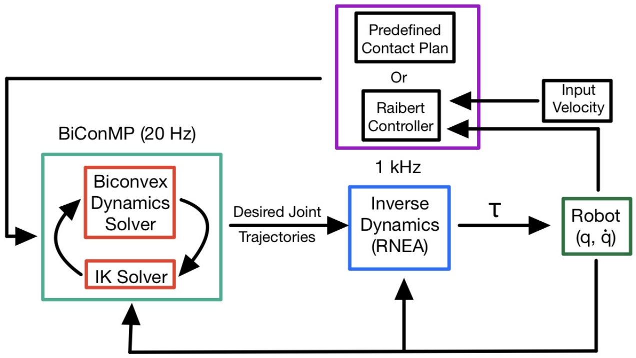

Fig. 1: A birds eye view of the entire nonlinear MPC framework. First, centroidal trajectories are generated using ADMM framework. These trajectories are used within a DDP-based kinematic optimizer that generates the desired joint trajectories. The optimal force and joint trajectories from this kino-dynamic iteration are recomputed every 50 ms and are used in an unconstrained inverse dynamics to compute the desired joint torques at 1 KHz. Finally these actuator torques are summed up with a fixed low impedance joint controller that result in the torques sent to the robot actuators.

to wall time which allows us to exploit the aforementioned convergence properties and make it attractive for use in an MPC fashion. In the proposed ADMM formulation, each convex sub-problem is solved using a custom implementation of the Fast Iterative Shrinkage Thresholding Algorithm (FISTA). This approach guarantees quadratic convergence of the convex sub-problems while enforcing a variety of constraints including second order friction cone constraints.

The nonlinear inverse kinematics (IK) sub-problem is solved using DDP. This removes the need to specify heuristic-based end-effector trajectories (e.g. via a spline based swing foot trajectory) as is often done in MPC implementations [2]. Although these methods of swing-foot generation work well for simple motions, they are often be restrictive in nature and do not allow the algorithm the freedom to find trajectories which may utilize the full capabilities of the end-effectors. The nonlinear IK often generates non-trivial swing foot trajectories to track the desired centroidal momentum provided by the dynamics optimization.

Given the desired forces from the centroidal MPC and joint trajectories from DDP-based kinematic solver, we compute the joint torques through an unconstrained inverse dynamics. This whole framework enable us to perform 20 Hz whole-body MPC on a quadruped robot where we send directly the computed joint torques to the robot without having a constrained QP-based inverse dynamics on top of the MPC. This highlights the quality of the trajectories.

An overview of the entire framework is shown in Fig. 1. Given the current states of the robot (joint position, velocity and accelerations), desired gait, planning horizon and velocity, a contact plan is either generated and adapted using the Raibert controller or pre-defined without contact adaptation for acyclic or general motions. The BiConMP framework takes the input states and computes the optimal end effector forces, joint positions, joint velocities and joint acceleration trajectories for the entire horizon. Given the desired joint trajectories and contact forces, we use inverse dynamics (computed using recursive Newton Euler algorithm) along with a low joint impedance around the desired states to compute the desired torques at 1 KHz. The desired torques is then sent to the robot which are tracked on board at 1 KHz. The entire full body planning loop is re-planned at 20 Hz (50 ms) to update optimal trajectories.

The entire BiConMP is implemented in C++. The biconvex dynamics optimizer is implemented from scratch. Croccoddyl is used in C++ to compute the IK solutions. The efficient C++ implementation of the BiConMP makes it possible to be used real-time MPC framework on the real robot. For more details regarding the implementation of the BiconMP framework we refer the reader to [1].

III. Experiments

1) Real robot motions: - We generate different gaits such as trots, jumps and bounds for the Solo12 quadruped and some snapshots are depicted in Fig. 2. The BiConMP is able to track desired linear and angular velocities accurately on the robot irrespective of the gait. The motions are also robust to unforeseen disturbances (push recovery) and unaccounted uneven terrain. In addition, we are able to demonstrate unplanned stable transitions between the 3 different gaits by taking advantage of the online re-planning capability of the framework. All these experiments are shown in the accompanying video [1].

To demonstrate the capability of the BiconMP to generate arbitrary dynamic acyclic trajectories online, we perform a High-Five motion on Solo12. The goal for the robot is to give a High-Five to a person in front of it by first raising both its front legs at a height above its base. Then, the robot must reach one of its arms out forward in order to High-Five at a fixed position. To generate this motion, the BiconMP framework is provided a contact plan for the feet, the desired location of the hand to be Hi-Fived and the time when the leg should reach the hand. The framework is then able to generate the forces and swing foot trajectories that can achieve this behaviour. Figure 3 shows the High-Five motion generated on the robot. During this motion, we would like to highlight that the inverse kinematics sub-problem automatically finds a non-trivial rotating motion for the front legs after performing the High-Five in order to track the centroidal momentum trajectories (i.e. momentum-obeying) that is provided by the biconvex centroidal dynamics solver. Specifically, the front legs of the robot swing their legs backwards and pivot around the hip joint in order to obey the momentum profile. Such a swing foot trajectory is very difficult to design apriori for a given motion which highlights the advantages of the whole-body motion planner.

[1] Video link - https://www.youtube.com/watch?v=Oz5eYBGoiok

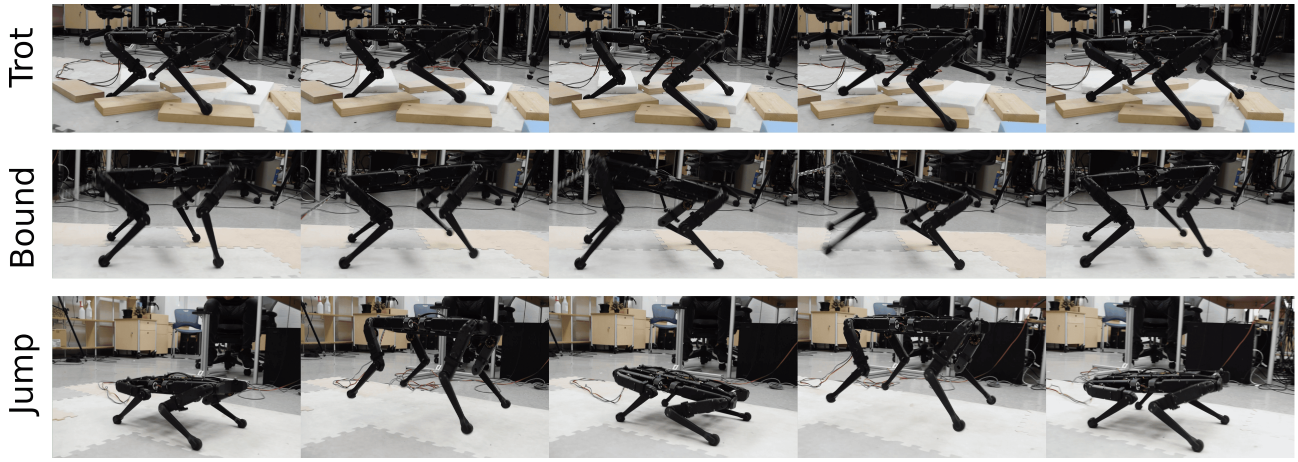

Fig. 2: Different motions demonstrated on real robot with solo12.

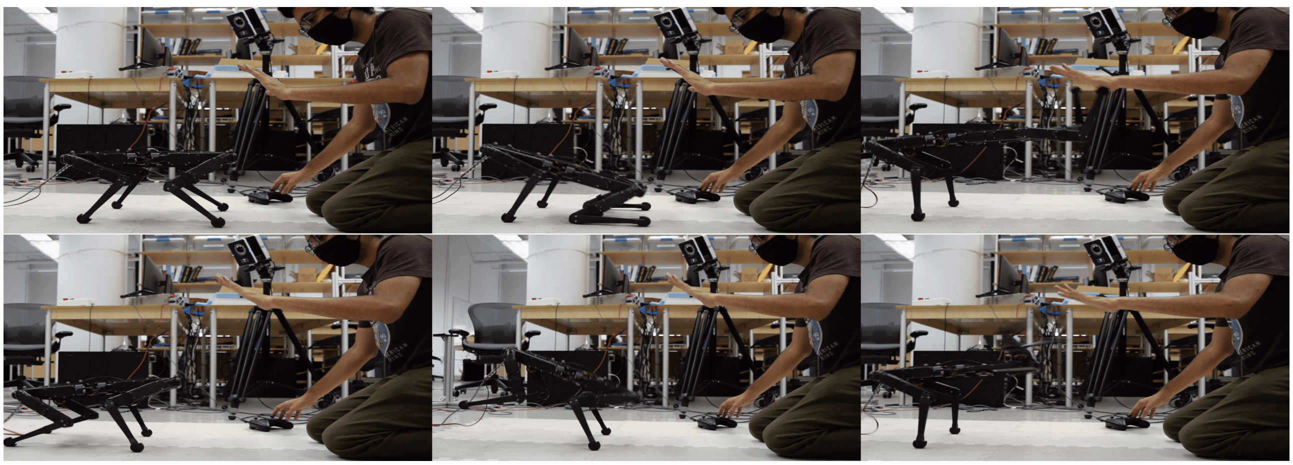

Fig. 3: Acyclic motions.(Top left to bottom left in clock wise direction)

2) MPC Analysis: Using the BiConMP framework, we perform an extensive analysis on the advantages gained by online replanning on the real robot. Further, the impact of the horizon and replanning frequency on the robustness and stability of the MPC framework are also studied. Briefly the obtained results are :

- we observe that by replanning online allows direct sim to real transfer without any gain tuning. Also, the same small impedance gains works across different motions because the BiConMP does most of the force control. This is usually not the case with offline trajectory optimization.
- Increasing the replanning frequency improves robustness to reject external disturbances and terrain in general. However, this gain starts to diminish after 100Hz.
- At low replanning frequencies, increasing the horizon improves the stability of the MPC and its capability to navigate external pushes & terrain safely.

For more details about the analysis and experimental setup we refer the reader to [1] .

CONCLUSION

In this work, we proposed a novel nonlinear MPC framework, the BiConMP, which is capable of generating whole body motion plans in real-time for various legged robots. We close the loop by implementing this framework in an MPC fashion to generate several motions such as trotting, bounding, and jumping on Solo12. In addition, we show that we are able to successfully generate arbitrary motions such as a High-Five which leverage the full-body capabilities of the robot. Finally, we briefly present the MPC analysis on the real robot.

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